HYDRODYNAMIC INTERACTION OF CASCADES OF PLATES DURING THEIR RELATIVE MOTION

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The problem of the hydrodynamic interaction taking place between the profiled plates of a (turbine) cascade is considered. This problem has hitherto been little studied – existing papers [1, 2] consider the model of effectively steady-state flow, the flow of the liquid at any instant of time being regarded as free from eddies, while time plays the part of a parameter; however, under practical conditions the field of velocities around moving cascades changes rapidly with time, so casting doubt on the validity of the steady-flow mode. The model proposed in this paper allows for high-frequency pulsations in the flow. The approximate solution here derived determines the interference between the profiles on the basis of a limiting solution with infinitely large Strouhal numbers. A typical calculation is presented.

1. Let us consider the flow of an ideal, incompressible liquid around a system of two cascades of thin profiles (plates); the velocity of the liquid a long way from the plates is v_1. Let the second cascade move along the front of the first at a steady velocity u. We denote the profiles of each cascade by the numbers $j = 0, \pm 1, \ldots$. Let us introduce a system of inertial Cartesian coordinates $Ox_{rj}y_{rj}$ associated with each j-th rigid profile of the r-th cascade (r = 1, 2), choosing the origin of coordinates in the middle of the chords of the profiles. We use h_r, c_r, and β_r to denote the step, half-chord, and stagger angle of the r-th cascade, d the axial gap between the cascades, l_0 the distance between the origins of the coordinate systems $Ox_{10}y_{10}$ and $Ox_{20}y_{20}$ at the instant of time t = 0 (Fig. 1). We consider that the geometrical parameters of the cascade satisfy the equation

$$h_1N_1 = h_2N_2 = L \tag{1.1}$$

where L is the smallest distance in which a whole number of profiles of the first and second cascades may be packed, N_1 and N_2 are natural numbers. Then in the case of rigid profiles the flow of liquid changes periodically with a period T = L/u in time and in coordinates with a period L along the front of the cascades. This period remains intact for the case of vibrating profiles if the angular frequency of the vibrations ω is chosen as

$$\omega = T / (2n\pi), \quad n = 1, 2, \ldots \tag{1.2}$$

Let us assume that the flow of the liquid outside the profiles and eddy tracks is of the potential type, the perturbations introduced into the flow by the profiles slight, while the Strouhal numbers

$$k_r = \Omega_r c_r / V_r \gg 1, \quad r = 1, 2; \quad \Omega_r = 2\pi N_r / T \tag{1.3}$$

where V_1, V_2 are the relative velocities of the main flow in front of the first and second cascades.

For the velocity potential $\varphi(x, y, t)$ of the perturbed motion of the liquid around the cascades of vibrating profiles we may formulate the following boundary problem ($x = x_{10}$, $y = y_{10}$). The function $\varphi(x, y, t)$ outside the profiles $|x_{rj}| \leq c_r$, $y_{rj} = \varepsilon_{rj}$ and eddy tracks L_{rj}($r = 1, 2$; $j = 0, \pm 1, \ldots$) satisfies the Laplace equation

$$\Delta\varphi = 0 \tag{1.4}$$

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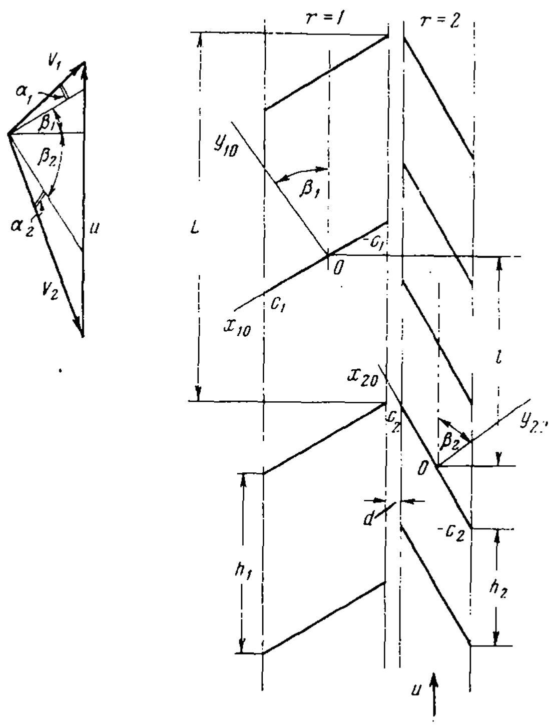

Fig. 1

the equation which states that the liquid cannot flow through the profile

$$(\mathbf{v}_1 + \nabla\varphi)\cdot\mathbf{y}_{rj} = \partial\varepsilon_{rj}/\partial t + (\mathbf{u}\cdot\mathbf{y}_{rj})\,\delta_{r2} \quad \text{for} \quad y_{rj} = \varepsilon_{rj},\ |x_{rj}| \leqslant c_r \tag{1.5}$$

the condition of pressure continuity in the eddy tracks

$$[p] = 0 \quad \text{for} \quad (x,\ y) \in L_{rj} \tag{1.6}$$

the conditions representing the attenuation of the perturbed velocities at an infinite distance in front of the cascades and periodicity of the flow in time t and coordinates x, y

$$\lim_{R\to\infty} \nabla\varphi = 0, \qquad R = \sqrt{x^2 + y^2}$$
$$\varphi(x,\ y,\ t) = \varphi(x,\ y,\ t + T), \quad \varphi(x,\ y,\ t) = \varphi(x - L\sin\beta_1,\ y + L\cos\beta_1,\ t) \tag{1.7}$$

and the Zhukovskii postulate on the sharp trailing edges of the profiles.

Here $\mathbf{y}_{rj}$ is the unit vector of the y_{rj} axis, ε_{rj} is the normal component of the deformation of the j-th profile of the r-th cascade during its vibrations, δ_{r2} is the Kronecker symbol, and p is the hydrodynamic pressure. The contours L_{rj} are considered as deviating very little from the straight lines $y_{rj} = 0$, $x_{rj} < c_r$.

We determine the velocity potential φ by the interference method [3] in the form

$$\varphi(x,\ y,\ t) = \sum_{n=1}^{2}\sum_{\sigma=-\infty}^{\infty} \varphi_{n\sigma}(x_{n\sigma},\ y_{n\sigma},\ t) \tag{1.8}$$

where $\varphi_{n\sigma}$ is the velocity potential for flow around a single profile with the nonpenetration condition

$$\partial\varphi_{n\sigma}/\partial y_{n\sigma} = v_{n\sigma}(x_{n\sigma},\ t) + u_{n\sigma}(x_{n\sigma},\ t) \quad \text{for} \quad y_{n\sigma} = \varepsilon_{n\sigma},\ |x_{n\sigma}| \leqslant c_n \tag{1.9}$$

Here $v_{n\sigma}$ is the specified normal component of the velocity of the profile

$$v_{n\sigma} = \partial\varepsilon_{n\sigma}/\partial t + (\delta_{n2}\mathbf{u} - \mathbf{v}_1)\cdot\mathbf{y}_{n\sigma} \tag{1.10}$$

while $u_{n\sigma}$ is an unknown function which is introduced in order to satisfy conditions (1.5) on all the profiles.

In accordance with the periodicity conditions

$$v_{n\sigma} = v_{n,\sigma+N_n}, \quad u_{n\sigma} = u_{n,\sigma+N_n}, \quad n = 1,\ 2 \tag{1.11}$$

Let us express the functions $v_{n\sigma}$, $u_{n\sigma}$ in the form

$$\begin{aligned} v_{n\sigma} &= {}^1\!/_2 q_0^{(n\sigma)} + \sum_{m=1}^{\infty} q_m^{(n\sigma)} \cos m\eta_{n\sigma}, \\ u_{n\sigma} &= {}^1\!/_2 \delta_0^{(n\sigma)} + \sum_{m=1}^{\infty} \delta_m^{(n\sigma)} \cos m\eta_{n\sigma} \end{aligned} \tag{1.12}$$

where $\eta_{n\sigma} = \operatorname{arc\,cos}\,(x_{n\sigma}\ c_n)$, $q_m^{(n\sigma)}$ are known and $\delta_m^{(n\sigma)}$ unknown coefficients. Substituting (1.8) into (1.5) and allowing for (1.9)-(1.12), in order to determine the coefficients $\delta_m^{(n\sigma)}$ we obtain a system of $(N_1 + N_2)$ linear equations

$${}^1\!/_2 \delta_0^{(rj)} + \sum_{m=1}^{\infty} \delta_m^{(rj)} \cos m\eta_{rj} + \sum_{\substack{\lambda=-\infty \\ (\lambda\neq 0)}}^{\infty} \sum_{\varkappa=1}^{N_r} \frac{\partial \varphi_{r\sigma}}{\partial y_{rj}} + \sum_{\lambda=-\infty}^{\infty} \sum_{\varkappa=1}^{N_n} \frac{\partial \varphi_{np}}{\partial y_{rj}} = 0 \tag{1.13}$$

for $y_{rj} = \varepsilon_{rj}$, $|x_{rj}| \leq c_r$; $n, r = 1, 2$; $n \neq r$; $j = 1, \ldots, N_r$; $\sigma = \lambda N_r + \varkappa$, $p = \lambda N_n + \varkappa$.

Here the coordinates x_{rj}, y_{rj} are related to the coordinates $x_{r\sigma}$, $y_{r\sigma}$, and x_{np}, y_{np} by the equations

$$\begin{aligned} x_{r\sigma} = x_{rj} + (\sigma - j)\, h_r \sin\beta_r, \quad & y_{r\sigma} = y_{rj} - (\sigma - j)\, h_r \cos\beta_r \quad \text{for} \quad n = r \\ x_{n\sigma} = x_{n0} + \sigma h_n \sin\beta_n, \quad & y_{n\sigma} = y_{n0} - \sigma h_n \cos\beta_n \quad \text{for} \quad n \neq r \end{aligned} \tag{1.14}$$

$$\begin{aligned} x_{n0} &= x_{r0} \cos(\beta_n - \beta_r) + \operatorname{sign}(n - r)\,[y_{r0} \sin(\beta_n - \beta_r) - l \sin\beta_n + h \cos\beta_n] \\ y_{n0} &= y_{r0} \cos(\beta_n - \beta_r) + \operatorname{sign}(n - r)\,[-x_{r0} \sin(\beta_n - \beta_r) + l \cos\beta_n + h \sin\beta_n)] \end{aligned} \tag{1.15}$$

where x_{r0}, y_{r0} are calculated from (1.14) and

$$l = l_0 + ut, \quad h = c_1 \cos\beta_1 + c_2 \cos\beta_2 + d$$

Let us express the velocity potential $\varphi_{n\sigma}$ in the form

$$\varphi_{n\sigma}(x_{n\sigma},\ y_{n\sigma},\ t) = \varphi_{n\sigma}^{(0)}(x_{n\sigma},\ y_{n\sigma}) + \varphi_{np}^{(1)}(x_{n\sigma},\ y_{n\sigma},\ t) \tag{1.16}$$

where $\varphi_{n\sigma}^{(0)}$, $\varphi_{n\sigma}^{(1)}$ are the steady and transient parts of $\varphi_{n\sigma}$. As $\varphi_{n\sigma}^{(0)}$ we may take the value of the function $\varphi_{n\sigma} = \varphi_{n\sigma}^*$, obtained by solving the problem (1.4)-(1.7) in a quasisteady formulation, averaged over the period T. Then

$$\varphi_{n\sigma}^{(0)}(x_{n\sigma}, y_{n\sigma}) = \frac{1}{T} \int_0^T \varphi_{n\sigma}^*(x_{n\sigma}, y_{n\sigma}, t)\, dt \tag{1.17}$$

The function $\varphi_{n\sigma}^*$ is determined by the equations

$$\varphi_{n\sigma}^* = \Phi_{n\sigma}^{(0)}(x_{n\sigma}, y_{n\sigma}, t) + \int_{\infty}^{x_{n\sigma}} \frac{\partial}{\partial y_{n\sigma}} g_{n\sigma}(x, y_{n\sigma})\, dx \tag{1.18}$$

$$\Phi_{n\sigma}^{(0)} = \sum_{m=1}^{\infty} \frac{1}{2m} [q_{m-1}^{(n\sigma)} - q_{m+1}^{(n\sigma)} + r_{m-1}^{(n\sigma)} - r_{m+1}^{(n\sigma)}]\ \mathrm{Im}\,\{z_{n\sigma} - \sqrt{z_{n\sigma}^2 - 1}\} \tag{1.19}$$

$$g_{n\sigma} = -{}^1\!/_2 [q_0^{(n\sigma)} - q_1^{(n\sigma)} + r_0^{(n\sigma)} - r_1^{(n\sigma)}]\ \mathrm{Re}\,\{\ln[z_{n\sigma} - \sqrt{z_{n\sigma}^2 - 1}]\} \tag{1.20}$$

Here $\Phi_{n\sigma}^{(0)}$ is the velocity potential of irrotational flow around the corresponding profile, the function $g_{n\sigma}$ defines the additional quasi-steady flow due to the satisfaction of the Zhukovskii postulate at the trailing edge of the same profile, $z_{n\sigma} = x_{n\sigma} + iy_{n\sigma}$, the coefficients $r_m^{(n\sigma)}$ are the Fourier coefficients for the function $u_{n\sigma}^*$, allowing for the interference of the other profiles. The quantities $r_m^{(n\sigma)}$ are determined as a solution of the system (1.13) in which $\delta_m^{(n\sigma)} = r_m^{(n\sigma)}$, $\varphi_{n\sigma} = \varphi_{n\sigma}^*$.

The velocity potential $\varphi_{n\sigma}^{(1)}$ of the additional transient flow should, in accordance with (1.9), (1.12), (1.16)-(1.20), satisfy the nonpenetration condition

$$\frac{\partial\varphi_{n\sigma}^{(1)}}{\partial y_{n\sigma}} = \frac{1}{2}[\theta_0^{(n\sigma)} + \delta_0^{(n\sigma)}] + \sum_{m=1}^{\infty}[\theta_m^{(n\sigma)} + \delta_m^{(n\sigma)}]\cos m\eta_{n\sigma}$$
$$\text{for} \quad y_{n\sigma} = \varepsilon_{n\sigma}, \quad |x_{n\sigma}| \leqslant c_n \tag{1.21}$$

where $\delta_m^{(n\sigma)}$ are unknown coefficients

$$\theta_m^{(n\sigma)} = q_m^{(n\sigma)} + r_m^{(n\sigma)} - \frac{1}{T}\int_0^T [q_m^{(n\sigma)} + r_m^{(n\sigma)}]\,dt \tag{1.22}$$

We determine the coefficients $\delta_m^{(n\sigma)}$ as the solution of the system (1.13) for $\varphi_{n\sigma} = \varphi_{n\sigma}^{(1)}$. The functions $\varphi_{n\sigma}^{(1)}$ are assumed equal to the velocity potential of irrotational flow around the profile $\Phi_{n\sigma}^{(0)}$. This approximation is based on the assumption (1.3) and the fact that in the limiting case of infinitely large Strouhal numbers the hydrodynamic influence of the other profiles in the cascade on the one under consideration is determined simply by the irrotational part of the flow of liquid [3]. In the approximation under consideration the presence of eddy tracks is only allowed for in the problem of flow around each individual profile, while the interaction between the profiles (cascades) is considered without allowing for the tracks. The system of equations (1.13) is solved by the method of collocations.

2. The hydrodynamic pressure p_{kj} at the points of the j-th profile of the k-th cascade is determined by the Cauchy–Lagrange integral

$$p_{kj} - p_0 = -\rho\,(\partial\varphi / \partial t - V_k \partial\varphi / \partial x), \quad |x_{kj}| \leqslant c_k, \; y_{kj} \pm 0, \; p_0 = \text{const} \tag{2.1}$$

The lifting force Y_{kj} and the moment M_{kj} are given by

$$Y_{kj} = \int_{-c_k}^{c_k} [p_{kj}(x, -0, t) - p_{kj}(x, +0, t)]\,dx$$
$$M_{kj} = \int_{-c_k}^{c_k} x\,[p_{kj}(x, -0, t) - p_{kj}(x, +0, t)]\,dt \tag{2.2}$$

Here ρ is the density and M_{kj} the moment relative to the middle of the profile. At the points of the profile under consideration only the velocity potential φ_{kj} suffers a discontinuity. Hence Eq. (2.2) with due allowance for (1.16)-(1.21) may be written in the form

$$Y_{kj} = -\pi\rho V_k c_k \Big\{\frac{1}{T}\int_0^T [q_0^{(kj)} - q_1^{(kj)} + r_0^{(kj)} - r_1^{(kj)}]\,dt +$$
$${}^1\!/_2\,[\theta_0^{(kj)} - \theta_1^{(kj)} + \delta_0^{(kj)} - \delta_1^{(kj)}] +$$
$$(c_k / 4V_k)(\partial / \partial t)\,[\theta_0^{(kj)} - \theta_2^{(kj)} + \delta_0^{(kj)} - \delta_2^{(kj)}]\Big\} \tag{2.3}$$

$$M_{kj} = -\frac{\pi}{2}\rho V_k c_k^2 \Big\{\frac{1}{T}\int_0^T [q_0^{(kj)} - q_2^{(kj)} + r_0^{(kj)} - r_2^{(kj)}]\,dt +$$
$${}^1\!/_2\,[\theta_0^{(kj)} + \theta_1^{(kj)} + \delta_0^{(kj)} + \delta_1^{(kj)} - 2(\theta_2^{(kj)} + \delta_2^{(kj)})] +$$
$$(c_k / 4V_k)(\partial / \partial t)\,[\theta_1^{(kj)} - \theta_3^{(kj)} + \delta_1^{(kj)} - \delta_3^{(kj)}]\Big\} \tag{2.4}$$

By way of example, Figs. 2 and 3 present the results of a calculation of the hydrodynamic interaction between the two cascades shown in Fig. 1, having the parameters

$$\tau_1 = 0.8, \;\; \tau_2 = 1.2 \;\; (\tau_k = b_k / h_k, \;\; b_k = 2c_k), \;\; c_1 / c_2 = 1,$$
$$l_0 = 2c_1, \;\; d = 0.2c_1, \;\; \beta_1 = 30°, \;\; \beta_2 = -60°, \;\; \alpha_1 = -\alpha_2 = 0.2$$

Here α_k means the angle between the velocity vector V_k and the chord of the profiles of the k-th cascade. In Eq. (1.12) we allowed for six terms of the Fourier series, which enabled us to reduce (1.13) to a system of 30 algebraical equations. The time step was chosen as $\Delta t = T/20$. The results of the calculations are presented in the form of the aerodynamic coefficients of the lifting force $C_{yk} = Y_{k0}/({}^1\!/_2\rho V_k^2 b_k)$ and the

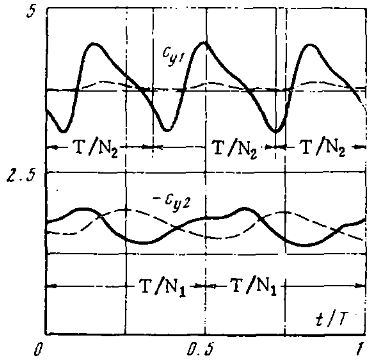

Fig. 2

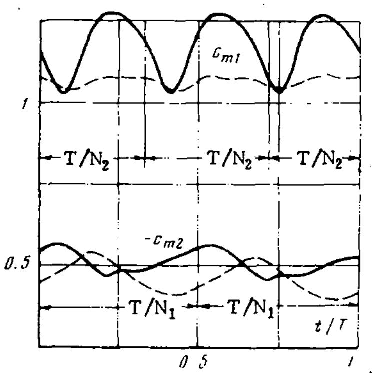

Fig. 3

moment $C_{mk} = M_{k0}/(^1/_2 \rho V_k^2 b_k^2)$ expressed as functions of t for the original profiles of the first and second cascades. The broken line corresponds to a calculation based on the hypothesis of a quasi-steady state, while the continuous curves are based on Eqs. (2.3) and (2.4). The arrows in Figs. 2 and 3 indicate intervals equal to T/N_2 (top) and T/N_1 (bottom).

In the case of the interaction of two cascades, the hydrodynamic interactions in the first cascade vary periodically in time with a period T/N_2 and in the second cascade with a period T/N_1 [2]. The results of the calculation obey this law, although in the algorithm the periodicity of the flow was only visualized as having a general period T. This constitutes one of the criteria for estimating the accuracy of the calculation. The algorithm in question may be extended to the case of hydrodynamic interaction between several cascades. In particular the program used for the computer calculation was set up for three cascades.

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